SDM College of Engineering & Technology, Dharwad-02

Department of Mathematics

Academic Year: 2022-2023.

Minutes of meeting

IQAC was held on 09/12/23 in the department of Mathematics, SDMCET, Dharwad to discuss the Internal Assessment question paper ~~I~~ / II / ~~III~~ of III semester UG in the Course Engg. Mathematics with Course Code 22UMAC300

The following were observed / discussed

1. Change Q.NO. 2(b) to Generating Function.
2. Mention alternate solution for question (1)(b) in Scheme
3.

Sl.No	Member's present	Signature
1	Dr. Jenifer K	[signature]
2	Dr. Basavaraj.H	[signature]
3	Prof. P. S. Badiger	[signature]
4		

9/12/23

Dr.Varsha D.Joshi

IQAC Co-ordinator

Dr.Jenifer J.Karnel

Chairman IQAC

Department of Mathematics

SDM COLLEGE OF ENGINEERING & TECHNOLOGY, DHARWAD

Department of Mathematics

Internal Assessment Test - II

Semester: III

Date: 13/12/2023

Course Title & Code: Engineering Mathematics-III (22UMAC300) **Time: 9-30 To 10-30 AM**

Course Teacher(s): JK, DPB, SSS, VJ, BH, PSB & PBJ. **Max Marks: 20**

Note: Question 3 is compulsory. Also answer any one full question from Q1 and Q2.

			Marks/COs
Q1	a	Find the Z-transform of $\sin(3n+5)$.	05M/CO2
	b	For a Bessel's function $J_n(x)$, prove that $2n\left[J_n(x)\right] = x\left[J_{n+1}(x) + J_{n-1}(x)\right]$	05M/CO3
Q2	a	Given $Z_T[u_n] = \frac{2z^2+3z+4}{(z-3)^3}, \lvert z\rvert > 3$. Show that $u_1 = 2$ and $u_2 = 21$.	05M/CO2
	b	Prove that $\int_0^1 x\left[J_n(\alpha x)\right]^2 dx = \frac{1}{2}\left[J_n'(x)\right]^2$. Change question to Generating fn.	05M/CO3
Q3	a	Solve the difference equation by using the Z-transform: $y_{n+2} - 4y_n = 0$, given that $y_0 = 0$ & $y_1 = 2$.	05M/CO2
	b	If $J_n(x)$ represents Bessel's function, then prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.	05M/CO3

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1. Dr. Jenifer. K Jenifer 9-12-23
2. Prakash S Badiger PSB 9-12-23
3. Dr. Varsha. J VP 9/12/23.

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		Marks/COs
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b	For a Bessel's function $J_n(x)$, prove that $2n\,[J_n(x)] = x[J_{n+1}(x) + J_{n-1}(x)]$.	05M/CO3
Q2a	Given $Z_T[u_n] = \frac{2z^2+3z+4}{(z-3)^3}, \lvert z\rvert > 3$. Show that $u_1 = 2$ and $u_2 = 21$.	05M/CO2
b	If $J_n(x)$ represents Bessel's function, then prove that $e^{\frac{x}{2}\left[t-\frac{1}{t}\right]} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$.	05M/CO3
Q3a	Solve the difference equation by using the Z-transform: $y_{n+2} - 4y_n = 0$, given that $y_0 = 0$ & $y_1 = 2$.	05M/CO2
b	If $J_n(x)$ represents Bessel's function, then prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.	05M/CO3

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Department of Mathematics

Internal Assessment Test – II

Solutions and Scheme

Semester: III

Course Title & Code: Engineering Mathematics-III (22UMAC300) **Date: 13/12/2023** **Time: 9-30 To 10-30 AM**

Course Teacher(s): JK, DPB, SSS, VJ, BH, PSB & PBJ. **Max Marks: 20**

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1 (a) Z- Transform of $\sin(3n+5)$

$$Z_T[\sin(3n+5)] = Z_T[\sin 3n \cos 5 + \cos 3n \sin 5] \rightarrow 1 \text{ Mark}$$

$$= \cos 5 \cdot \frac{z \sin 3}{z^2 - 2z\cos 3 + 1} + \sin 5 \cdot \frac{z(z-\cos 3)}{z^2 - 2z\cos 3 + 1} \rightarrow 2 \text{ Marks}$$

$$Z_T[\sin(3n+5)] = \frac{z[z\sin 5 - \sin 2]}{z^2 - 2z\cos 3 + 1} \rightarrow 2 \text{ Marks}$$

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1 (b) To prove $2n J_n(x) = x[J_{n+1}(x) + J_{n-1}(x)]$ OR Marks to be given for Alternative methods.

We know that $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{n+2r}}{r!\,\Gamma(n+r+1)} \rightarrow 1$ Mark.

Multiply both side by $2n$ & $2n = 2(n+r) - 2r$, by simplifying, we get the required result. } → 4 Marks

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2 (a) Given $Z_T[u_n] = \frac{2z^2+3z+4}{(z-3)^3}, |z|>3$

$u_0 = \lim_{z\to\infty} [\bar{u}(z)] = 0 \rightarrow 1$ Mark and $u_2 = \lim_{z\to\infty} \{z^2[\bar{u}(z) - u_0 - u_1 z^{-1}]\}$

$u_1 = \lim_{z\to\infty} \{z[\bar{u}(z) - u_0]\} = 2$ → 2 Marks $\quad u_2 = 21. \rightarrow 2$ Marks

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2 (b) To prove that $e^{\frac{x}{2}[t-\frac{1}{t}]} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$

Consider $e^{\frac{x}{2}[t-\frac{1}{t}]} = e^{\frac{xt}{2}} \cdot e^{-\frac{x}{2t}} \rightarrow 1$ Mark

Expanding the above exponential terms and by comparing the coefficients of t^n, we get the required result. } → 4 Marks

[Or → Marks to be given for alternative methods].

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3 (a) Let $y_{n+2} - 4y_n = 0$ with $y_0 = 0$ & $y_1 = 2$

$Z_T[y_{n+2}] - 4 Z_T[y_n] = Z_T[0]$

$$\Rightarrow \bar{y}(z) = \frac{2z}{z^2-4} = \frac{2z}{(z-2)(z+2)} = A \cdot \frac{z}{(z-2)} + B \cdot \frac{z}{(z+2)} \rightarrow 2 \text{ Marks}$$

$A = 1/2$ & $B = -1/2$

$$y_n = \frac{1}{2}(2)^n - \frac{1}{2}(-2)^n = 2^{n-1} + (-2)^{n-1} \Big\} \rightarrow 3 \text{ Marks}$$

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3 (b) To prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

Let $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{n+2r}}{r!\,\Gamma(n+r+1)} \rightarrow 1$ Mark

put $n = 1/2$ & by simplifying using the gamma function properties, we get

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right] = \sqrt{\frac{2}{\pi x}} \sin x \Big\} \rightarrow 4 \text{ Marks}$$

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