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SDM COLLEGE OF ENGINEERING & TECHNOLOGY, DHARWAD

DEPARTMENT OF MATHEMATICS

Semester End Examination, January 2021

Course Code:18UMAC300 **Semester: III**
Course Title: Engineering Mathematics III
Duration : 3 hours **Max. Marks: 100**

Note: Answer any one full question from each unit.

Unit – I

Q1 **6 Marks**

a Find the Laplace transform of $e^{4t}\sin 2t.\cos t$

b A periodic function of period $\frac{2\pi}{w}$ is defined by $f(t)=\begin{cases} E\sin wt & 0\le t<\pi/w \\ 0 & \pi/w\le t<2\pi/w \end{cases}$ **7 Marks**

Where E and w are constants. Show that $L[f(t)]=\dfrac{EW}{(s^2+w^2)(1-e^{-\pi s/w})}$

c A voltage Ee^{-at} is applied at $t=0$ to a circuit of inductance L and resistance R. **7 Marks**
Show that the current at any time t is $\dfrac{E}{R-aL}\left[e^{-at}-e^{-\frac{Rt}{L}}\right]$, by applying Laplace transform.

OR

Q2 **6 Marks**

a Evaluate $L^{-1}\left[s\log\left(\dfrac{s+4}{s-4}\right)\right]$

b Prove that, if $L[f(t)]=\bar{f}(s)$ then $L[t^n f(t)]=(-1)^n\dfrac{d^n}{ds^n}\left[\bar{f}(s)\right]$, where n is positive integer. **7 Marks**

c Express the following in terms of Heaviside unit step function. Hence find the **7 Marks**
Laplace transforms $f(t)=\begin{cases} \cos t & 0<t\le\pi \\ 1 & \pi<t\le 2\pi \\ \sin t & t>2\pi \end{cases}$.

Unit – II

Q3 **6 Marks**

a Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0\le x\le 2$.

b Obtain the sine half range series of $f(x)=\begin{cases} \frac{1}{4}-x & in\ \ 0<x<\frac{1}{2} \\ x-\frac{3}{4} & in\ \ \frac{1}{2}<x<1 \end{cases}$ **7 Marks** c Obtain the Fourier series for the function $f(x)=\begin{cases}-\pi & in\ -\pi<x<0\\ x & in\ 0<x<\pi\end{cases}$. Hence deduce that the sum of squares of the reciprocal of the odd integers is equal to $\pi^2/8$. **7 Marks**

OR

Q4

a Find the cosine half range series of f(x) = x sinx in 0< x <π. Deduce that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}---=\frac{\pi-2}{4}$ **6 Marks**

b An alternating current after passing through rectifier has the form **7 Marks**

$$I=\begin{cases}I_0\sin\theta & for\ 0<\theta\le\pi\\ 0 & for\ \pi<\theta\le 2\pi\end{cases}$$

Where I_0 is the maximum current. Express I as a Fourier series in (0, 2π).

c The following data gives the variations of a periodic current over a period **7 Marks**

t sec	0	T/6	T/3	T/2	2T/3	5T/6	T
A amp	9	18.2	24.4	27.8	27.5	22	9

Find numerically the direct current part of the variable current and obtain the amplitudes up to the second harmonic.

Unit – III

Q5

a Show that $Z_T\left[\frac{1}{n!}\right]=e^{1/z}$. Hence find $Z_T\left[\frac{1}{(n+1)!}\right]$ and $Z_T\left[\frac{1}{(n+2)!}\right]$. **6 Marks**

b Find **7 Marks**

$i)\ Z_T(\cos n\theta)=\frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$ $\quad ii)\ Z_T(\sin n\theta)=\frac{z\sin\theta}{z^2-2z\cos\theta+1}$

c Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|\ge 1\end{cases}$ and hence find the value of **7 Marks**

$$\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\left(\frac{x}{2}\right)dx.$$

OR

Q6

a Find the Fourier cosine transform of $\frac{1}{1+x^2}$. **6 Marks**

b Find the inverse Z-transform of $\frac{3z^2+2z}{(5z-1)(5z+2)}$. **7 Marks**

c Solve the difference equation, using Z -transform $y_{n+2}+2y_{n+1}+y_n=n$ given that $y_0=0=y_1$. **7 Marks**

Unit – IV

Q7

a Applying Taylors series method, find y at $x=1.02$ correct to five decimal places $dy=(xy-1)dx$ and $y=2$ at $x=1$. 6 Marks

b Use modified Euler's method to find $y(20.2)$ given that $\frac{dy}{dx}=\log_{10}\left(\frac{x}{y}\right)$ with $y(20)=5$ taking h=0.2. 7 Marks

c Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx}=\frac{y-x}{y+x}$, $y(0)=1$, taking h=0.2 7 Marks

OR

Q8

a Use modified Euler's method to compute y(0.1) given that $\frac{dy}{dx}=x^2+y, \quad y(0)=1$ by taking h=0.05 considering the accuracy up to two approximations in each step. 6 Marks

b The following table gives the solution of $5xy'+y^2-2=0$. Find the value of y at x=4.5 using Milne's predictor and corrector formula. Use the corrector formula twice. 7 Marks

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

c Use fourth order Runge-Kutta method to solve $(x+y)\frac{dy}{dx}=1$, $y(0.4)=1$ at $x=0.5$ correct to four decimal places. 7 Marks

Unit – V

Q9

a Find the extremal of the functional $\int_a^b (x^2y'^2+2y^2+2xy)dx$. 6 Marks

b Compute y(0.1) given $\frac{d^2y}{dx^2}=y^3$ and $y=10, \frac{dy}{dx}=5$ at x=0 by Runge –Kutta method. 7 Marks

c Apply Milne's method to compute y(0.8) given that $\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values, 7 Marks

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

OR

Q10

a Find the extremal of the functional $\int_{x1}^{x2} \left(y^2 + y'^2 + 2ye^x\right)dx$. **6 Marks**

b Obtain the value of x and $\frac{dx}{dt}$ when t=0.1 given that x satisfies the equation $\frac{d^2x}{dt^2} = t\frac{dx}{dt} - 4x$ and $x = 3, \frac{dx}{dt} = 0$ when t=0 initially, using Runge-Kutta method of fourth order. **7 Marks**

c Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the valve of the dependent variable corresponding to the value 1.4 of the independent variable by applying Milne's method using the following data. **7 Marks**

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y'	2	2.3178	2.6725	3.0657

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Q. No.	1a	1b	1c	3a	3b	3c	5a	5b	5c	7a	7b	7c	9a	9b	9c
CO	1	1	1	2	2	2	3	3	3	4	4	4	5	4	4
Q. No.	2a	2b	2c	4a	4b	4c	6a	6b	6c	8a	8b	8c	10a	10b	10c
CO	1	1	1	2	2	2	3	3	3	4	4	4	5	4	4